COMPUTER MODELLING OF COMPLEX SYSTEMS

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Summary

Most stratigraphic forward modelling software simulates the response of a sedimentary system to externally imposed controls which are not affected by the sedimentation process. A good example is modelling of carbonate platform architectures resulting from given sea level curves. However, many important controls on sedimentation are altered by the sedimentation itself. Good examples of situations where such feedbacks occur are:-

(i) Salt tectonics and sedimentation.
(ii) Mixed clastic/carbonate systems.
(iii) Waves, currents and sedimentation.

Modelling these coupled systems is more challenging than modelling of simpler systems because of mismatched time scales and amplified numerical instability.

Introduction

The stratigraphic and structural architecture of the Earth's crust results from the combined influence of many physical, chemical and biological processes. This paper discusses methods for computer modelling of multiple processes operating simultaneously. Modelling of such concurrent processes presents special problems not occuring within simpler models.

Models combining salt tectonics and sedimentation are a good example of a simple coupled system. Fig 1 shows such a model in which a delta progrades across a buried salt body which then flows as a result of the differential loading.

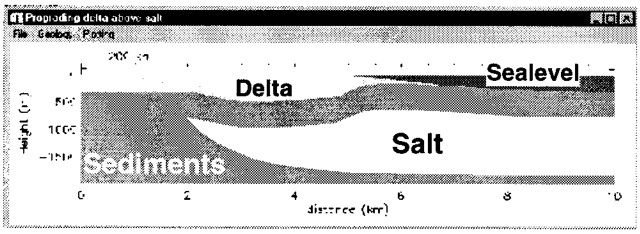

Fig 1. Delta prograding over a salt body.

In this model, the salt movement is affected by the sedimentation but, in addition, the salt movement changes the bathymettry hence affecting sedimentation. The principle results of this interaction are that the rate of progradation is reduced and a bump is generated in the sea floor ahead of the delta toe.

Geological time scales

Geologically relevant processes operate on time scales ranging from fractions of a second to hundreds of millions of years (table 1). The fastest such process is probably fault rupture propagation during earthquakes which occurs at speeds of several km per second. However, this does not directly contribute to the sedimentary record and will not be discussed further here. Turbidity currents and pyroclastic flows/surges (often triggered by earthquakes) are, perhaps, the next fastest and can flow at speeds approaching 100 ms^{-1}. Hence, if these are modelled with a resolution of 1m, a modelling time scale of 10^{-2} seconds is required. Surface water flow is generally much slower with typical speeds of the order of 1 ms^{-1} requiring modelling time scales around 1s. Aeolian processes occur at similar rates (or perhaps slightly faster) and require similar modelling time scales. Fluid flow through rocks, on the other hand, is significantly slower and very variable. Flow through an aquifer, for example, should occur at a rate similar to that at which it is recharged by rainfall (say 1 m/yr at maximum) requiring a modelling time scale of a year or so.

Interestingly, erosion, sedimentation, faulting and folding all occur at rates up to about 1 m/ka and therefore all need modelling time scales of around 1 ka. This is probably not a coincidence but, rather, is due to the fact that these all represent the Earth's response to the "space problems" created by plate tectonics which occurs at the slightly higher rate of around 1m per hundred years. This "coincidence" turns out to be very useful since it allows computer models to be built which simultaneously model tectonics, erosion and sedimentation.

A few processes occur at even slower rates. Diagenesis for example requires 10s of thousands of years to produce much change whilst compaction and thermal subsidence need 100s of thousands of years.

Process	Log(time scale)
Turbidity currents	-2
Pyroclastic flows/surges	-2
Wind	-1
River flow	0
Waves, tides, ocean currents	0
Fluid flow in rocks	7
Plate tectonics	9
Isostatic rebound	9
Erosion/deposition	10
Fault movement and folding	10
Salt diapirism	10
Diagenesis	11
Compaction	12
Thermal subsidence	12

Table 1. Approximate time scales required for modelling of various geological processes. Values are $\log_{10}$ of the time scale in seconds. A spatial resolution of 1m is assumed for most of these processes.

Detailed modelling of processes having such vastly different time scales is not possible and unlikely to become so for the foreseeable future. Instead, it is necessary to choose the most relevant modelling time scale and then find ways of approximating the effect of any included processes having very different time scales. This time-scale problem is the most obvious difficulty associated with modelling of multiple processes but other problems also occur even for processes having similar time scales.

Problems not caused by time-scale mismatch

(i) Coordinate systems:

All models presented in this paper concern finite difference implementations of partial differential equations although many of the conclusions also apply to other approaches (eg finite element models). For a finite difference model an important decision is whether to use an Eulerian or a Lagrangian coordinate system. An Eulerian implementation calculates model properties (e.g. layer thicknesses) at fixed locations which do not change with time and this is the natural framework for sediment deposition models. Thus, if the height $h(x,y)$ at a particular location in a model is changing at a rate $s(x,y)$ then the new height, after a time step of Δt is

$$h \rightarrow h + s.\Delta t \qquad (1)$$

where the new height is at an identical location to the old height.

In a Lagrangian model, on the other hand, calculations are made at points which move along with any advection and this is the most natural framework for models involving tectonics. Hence, if a point in the hangingwall to a fault is moving at rates u, v and w in the x, y and z directions respectively, then it's new location after one time step will be

$$\begin{aligned} x &\rightarrow x + u.\Delta t \\ y &\rightarrow y + v.\Delta t \\ z &\rightarrow z + w.\Delta t \end{aligned} \qquad (2)$$

i.e. the locations change with time. So, what framework should be used for a model combining tectonics and sedimentation ? In a Lagrangian model points initially close to one another can become widely separated as the model proceeds (e.g. consider points either side of a fault). This is very undesirable in a sedimentary model and so an Eulerian approach is prefered. An Eulerian model of tectonic deformation can be quite easily constructed using

$$d/dt = \partial/\partial t + u.\partial/\partial x + v.\partial/\partial y + w.\partial/\partial z \qquad (3)$$

where full derivatives represent differentiation in a Lagrangian framework and partial derivatives differentiation in an Eulerian framwork. Hence, the height of a given surface changes according to

$$\partial h/\partial t = w - u.\partial h/\partial x - v.\partial h/\partial y. \qquad (4)$$

Combining this with equation (1) then gives

$$h \rightarrow h + (s + w - u.\partial h/\partial x - v.\partial h/\partial y)\,.\Delta t. \qquad (5)$$

There are, unfortunately, two problems with this approach. Firstly, $h(x,y)$ must be single valued which restricts its use for cases of complex folding and overthrusting. Secondly, the procedure generates "numerical drag" adjacent to faults. The solution to both these problems is to use equation (5) for modelling the Earth's surface (which is usually single valued!) and equation (2) for modelling sub-surface points which, of course, have no sedimentation. Fig 2 shows and example output from such an algorithm resulting from a complex interaction of inversion tectonics, sea level fluctuation and variable sediment supply.

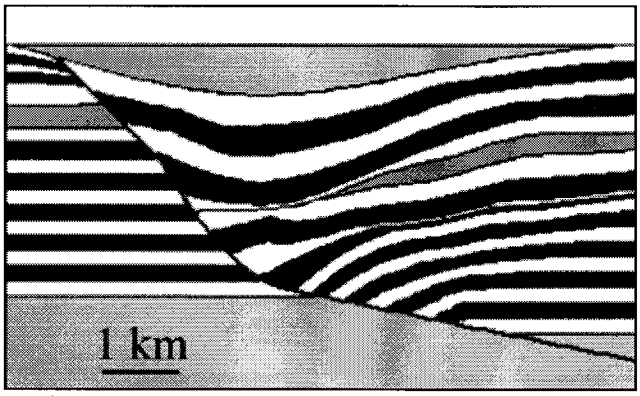

Fig 2. Tectonics and sedimentation example resulting from a complex interaction of inversion tectonics, sea level fluctuation and variable sediment supply.

(ii) Feedback Instability:

It often happens that algorithms which are numerically stable in isolation become unstable when combined. Fig 3 shows a simple example of this.

The top image shows the out put from a simple model in which depth-dependent carbonate deposition of the form

$$s = s_o\, exp(-z/Z_c) \qquad (6)$$

(where s is deposition rate, z is water depth and s_o and Z_c are constants) occurs onto a simple slope with a steadily rising sea level. The resulting platform is unable to prograde far into the basin but, most importantly, the result is numerically stable. The central image shows the same setting but, this time, the basin is filled by a prograding wedge of clastic sediment modelled using

$$s = -\partial F/\partial x = aF - e \qquad (7)$$

with

$$a = \text{const for } z > \text{wavebase}$$

and

$$e = e_o\, exp(-z/Z_e)$$

(where F is sediment flux, e is erosion rate and e_o and Z_e are constants). Again, the result is reasonable stable numerically although there is some slight noisyness which does not amplify with time.

The lowest image in fig 3 shows the result of simply combining the carbonate and clastic models. The model now produces unacceptably noisy surfaces. The reason for this is simply that the carbonate deposition algorithm amplifies the slight noise in the clastic results. Equation (6) implies that production decreases with depth, hence, any point which starts off slightly higher than its surroundings will have a slightly higher rate of carbonate growth and, hence, height differences become magnified. This includes height differences generated by slight noise in the clastic modelling algorithm.

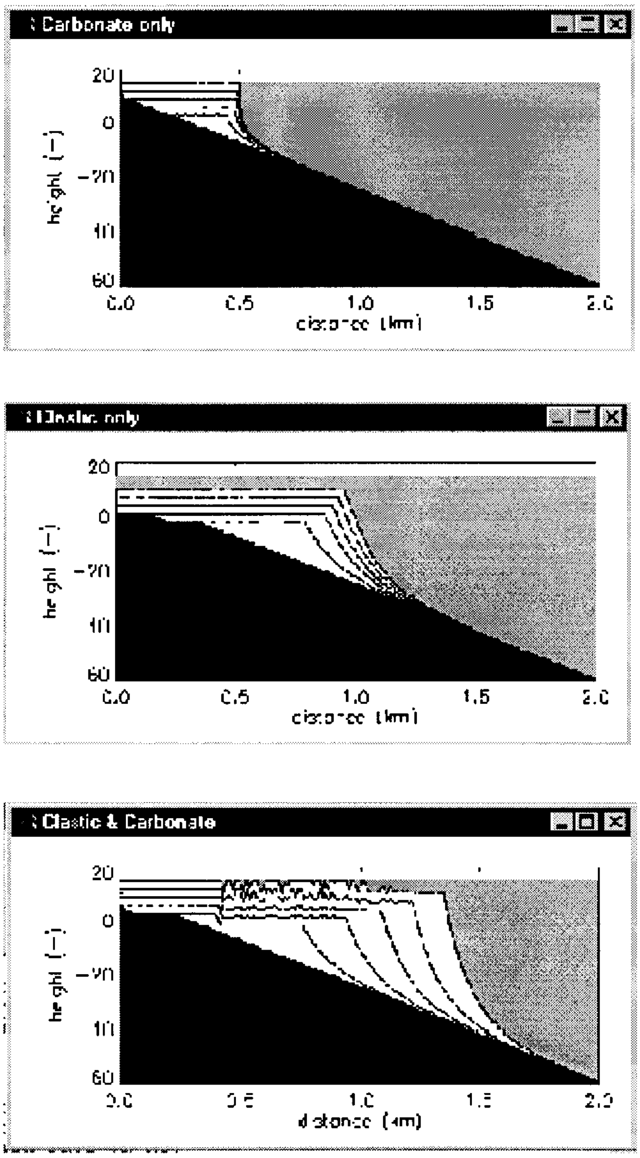

Fig 3. Simulations of carbonate deposition only (top) or clastic sedimentation only (centre) are reasonably smooth but the combination (bottom) is numerically unstable.

(iii) 2D versus 3D:

Many complex sedimentary systems can only be modelled in 3D since 2D models inherently miss out essential features of the interaction between different processes. Combined carbonate/clastic models are, again, a good example. Fig 4 shows such a model in which a plume of suspended sediment (origin at x=0, y=0 km) is transported in the positive y-direction by a long-shore current. The carbonate sedimentation rate is assumed to be reduced by increased depth (in this case governed by a uniform slope from sealevel at x=0 km and reduced by "poisoning" by clastic sedimentation. The equation used was

$$s = s_o \exp(-z/Z_c) \exp(-c/C) \qquad (8)$$

where c is clastic sedimentation rate and C is a constant. The point about this model is that it is possible to get areas of high carbonate productivity immediately offshore of areas with a high clastic sedimentation rate. This cannot be done in a 2D model.

It is also worth noting that the instability discussed in the previous sub-section is much less of a problem now since carbonate sedimentation tends to occur in different areas of the model from the clastic sedimentation.

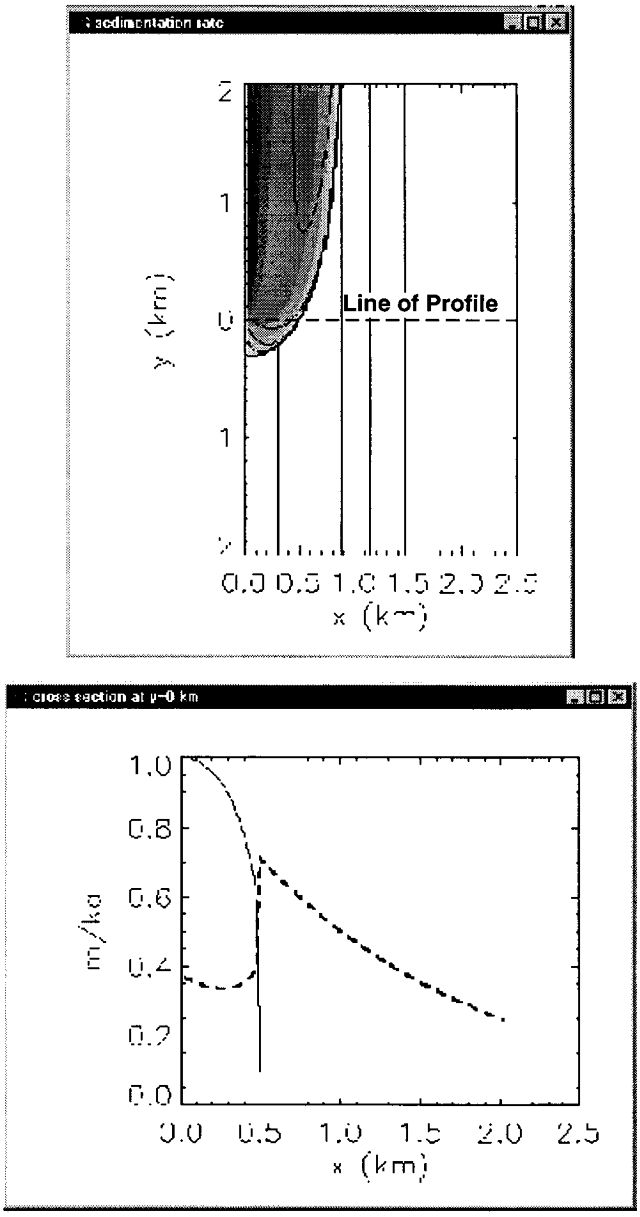

Fig 4. 3D combined clastic and carbonate sedimentation model. Top shows plan-view contours of clastic sedimentation rate (filled contours) and carbonate sedimentation rate (line contours). Bottom shows profile through y=0 km with clastic sedimentation rate shown as a full line and carbonate sedimentation rate as a dashed line.

Time-scale mismatch problems

(i) Steady state modelling:

The appropriate time scale for modelling of deposition is of the order of 1 ka and so faster processes (eg currents) must be approximated somehow. For continuously operating processes, a steady-state assumption may be appropriate. For example, we can assume that the current is constant throughout each time step. This is precisely the assumption made in producing fig 4.

However, this is not generally a very good assumption. Tidally generated currents, for example, change several times a day and plumes such as that modelled in fig 4 are quite likely to undergo seasonal changes. Instead, a weighted sum of steady-states can be used. Fig 5 shows a model which is identical to that of fig 4 except that a gaussian distribution of river input fluxes has been assumed. This result was generated simply by performing a large number of steady state simulations of the plume and then calculating a weighted average for them. Note that the result is NOT the same as that shown in fig 4 even although the same mean river flux was used in both cases and all other parameters are identical.

(ii) Intermittent process modelling:
At the other extreme are processes which are very intermittent such as turbidity current deposits. Turbidity currents evolve on time scales of seconds, last a few hours and occur, perhaps, once a decade. These simply cannot be treated as steady state phenomema. In these cases it is not possible to avoid the use of multiple time-step sizes in the modelling. For example, a time step of 1 second might be used for modelling a single turbidity current and the resultant deposit could be used to estimate the rate of turbidite deposition within a model having time steps of, perhaps, hundreds of years.

Conclusions
When modelling complex coupled systems many problems need addressing which are not apparent in simpler models:-

1. Modelling must be performed at the appropriate time scale. In the model shown in fig 1, for example, the time scale may be controlled by the requirement to model salt flow of a particular viscosity and thickness or it may be controlled by the requirement to model a very high flux of sediment.
2. Algorithms should be designed, where possible, to elegantly combine different processes. For example, fig 2 shows a model in which modelling both sedimentation and tectonics in an Eulerian framework allows these very different processes to be very robustly combined.
3. Care should be taken to ensure that problems are not caused by the combination of, otherwise, stable algorithms. Fig 3, for example, shows how NOT to combine clastic and carbonate sedimentation into one algorithm.
4. Many interactions between different processes occur in an inherently 3D fashion and cannot be modelled in 2D (eg see figs 4 & 5).
5. Processes which occur much more rapidly than the modelling time scale are often, explicitly or implicitly, assumed to be in a steady state. However, it is usually better to model such processes as occuring in a distribution of different steady states each having a distinct probability of occurence.

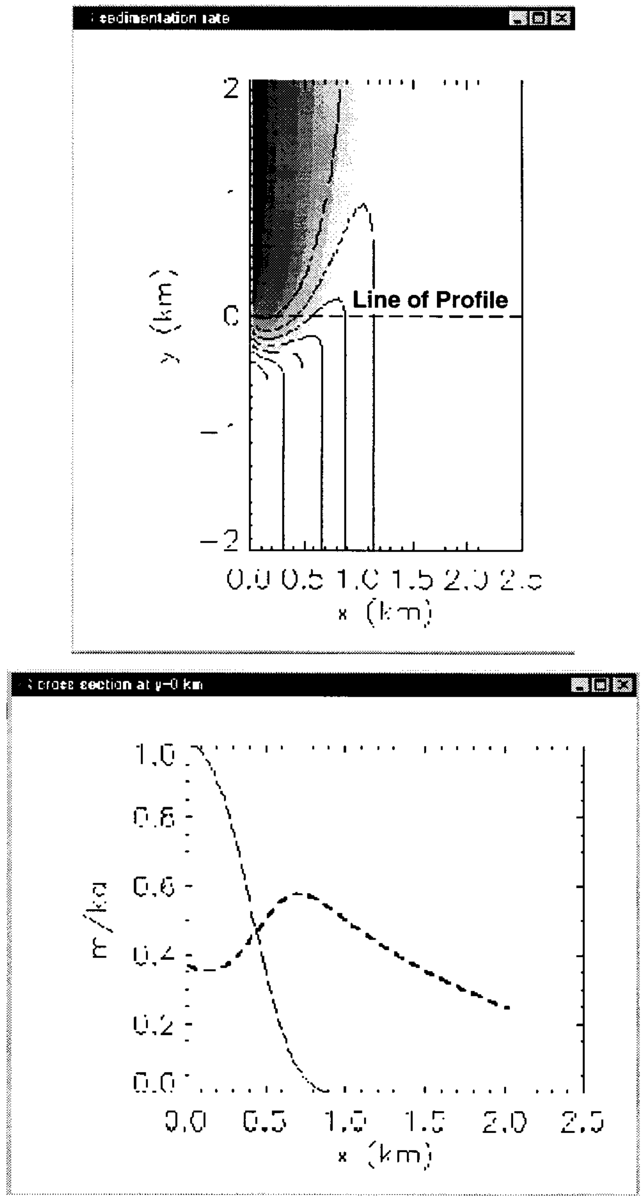

Fig 5. As for fig 4 but with the plume-generating river having a gaussian distribution of fluxes.

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